Frac-Turn #201

Practice 5C: Chapter 9, page 103, problem #201

201. ★★ Complete each of the following to solve the Frac-Turn puzzle below:

- Write each fraction on the left in its correct location on the outside of the grid.
- Place ***all ten digits*** (from 0-9) once each on the grid so that each fraction's path correctly traces its decimal form.

<u>Fractions</u>

$\frac{29}{40}$ $\frac{1}{6}$

$\frac{21}{4}$ $\frac{2}{11}$

$\frac{9}{4}$ $\frac{9}{11}$

$\frac{9}{10}$ $\frac{1}{15}$

$\frac{24}{5}$

•		❯		∨	•	∨
	❯				∨	
			•			∧
❮		•				
		∧				∧
	∧				❮	
∧				❮	•	•

Frac-Turn #201, Solution

Practice 5C: Chapter 5, page 103, problem #201

201. We convert each fraction to a decimal.

$\frac{29}{40} = .725$ $\quad\quad$ $\frac{1}{6} = .1\overline{6}$

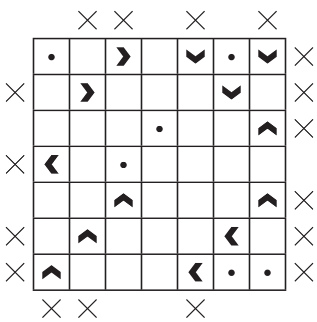
$\frac{21}{4} = 5.25$ $\quad\quad$ $\frac{2}{11} = .\overline{18}$

$\frac{9}{4} = 2.25$ $\quad\quad$ $\frac{9}{11} = .\overline{81}$

$\frac{9}{10} = .9$ $\quad\quad$ $\frac{1}{15} = .0\overline{6}$

$\frac{24}{5} = 4.8$

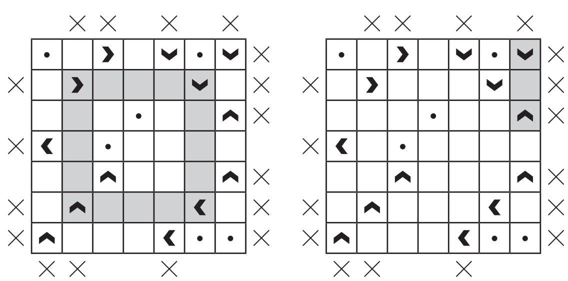
Then, we eliminate any location on the outside of the grid whose path does not include exactly one decimal point.



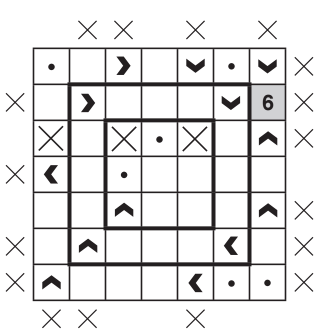
Then, we have four fractions with repeating digits:

$\frac{1}{6} = .1\overline{6}$, $\quad \frac{1}{15} = .0\overline{6}$, $\quad \frac{2}{11} = .\overline{18}$, $\quad \frac{9}{11} = .\overline{81}$.

The repeating digits in these fractions' paths must be contained within a loop. There are only two loops in the grid, as shown below.



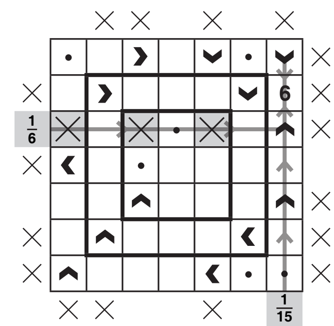
The smaller loop has only one empty square. Only the fractions $\frac{1}{6} = .1\overline{6}$ and $\frac{1}{15} = .0\overline{6}$ have a single repeating digit, so this loop must be part of both paths. Therefore, the empty square in this loop contains a 6.



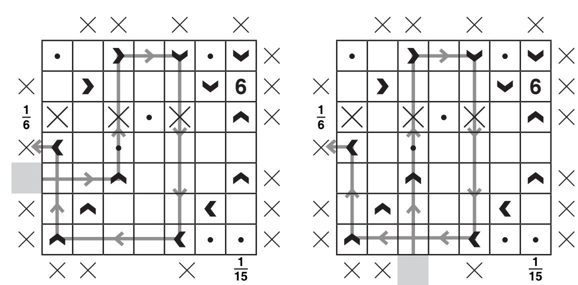
The larger loop must be part of the paths for $\frac{2}{11} = .\overline{18}$ and $\frac{9}{11} = .\overline{81}$. So, there is exactly one 1 and one 8 within this loop, and no other digits. Therefore, the 1 in the path for $\frac{1}{6} = .1\overline{6}$ lies somewhere in this larger loop.

There is only one path that includes squares from both the larger and smaller loops. We write $\frac{1}{6}$ at the beginning of this path, and place $\times$'s in the squares not inside the larger loop, since these squares cannot contain a 1.

Then, there is only one other path that includes the smaller loop, so we write $\frac{1}{15}$ at the beginning of that path.

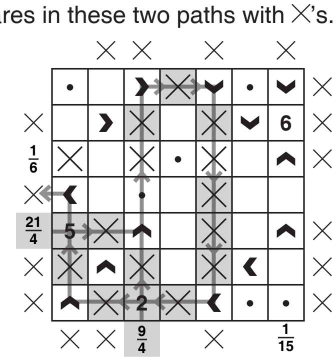


Each of $\frac{21}{4} = 5.25$ and $\frac{9}{4} = 2.25$ have two of the same digit in their path. Since we can only place each digit once, each of these paths must cross itself. There are only two paths that cross themselves.

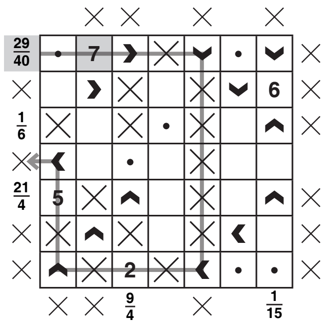


In the left path above, the cross-over occurs at the very first and last square of the path. So, this must be the path for $\frac{21}{4} = 5.25$. Then, the right path must be for $\frac{9}{4} = 2.25$.

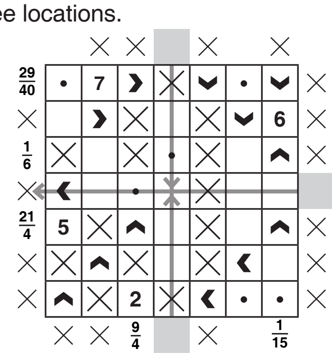
Placing the 5 and 2 at the two cross-over points completes the paths $\frac{21}{4} = 5.25$ and $\frac{9}{4} = 2.25$. There can be no other digits in these paths, so we fill the remaining empty squares in these two paths with $\times$'s.



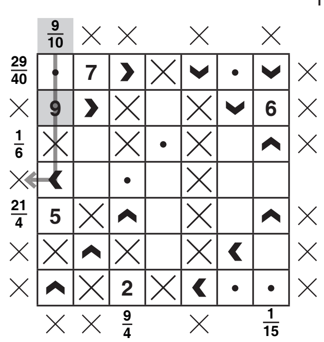
The path for $\frac{29}{40} = .725$ also contains the digits 2 and 5. There is only one remaining path that crosses the 2 and the 5, so we fill the squares below as shown.



The only two non-repeating fractions left to place are $\frac{9}{10}$ and $\frac{24}{5}$, and there are four remaining locations outside of the grid. Three of these locations begin paths that intersect at the center square. Since the paths for $\frac{9}{10} = .9$ and $\frac{24}{5} = 4.8$ do not share any digits, we can only use one of these three locations.

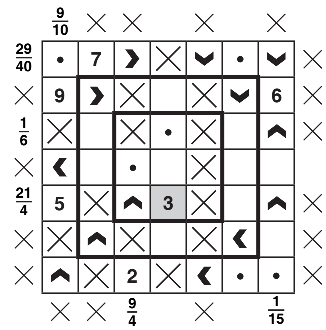


Therefore, the fourth location at the top-left of the grid must be used, and it can only be filled with $\frac{9}{10} = .9$.

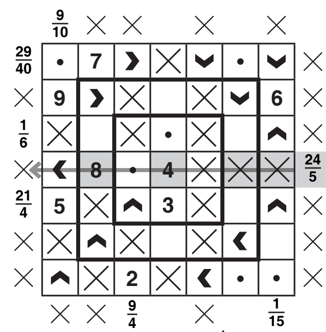


Since we must use all ten digits, and the digit 3 is not a part of any path, we must place a 3 somewhere so that it does not interfere with any other path.

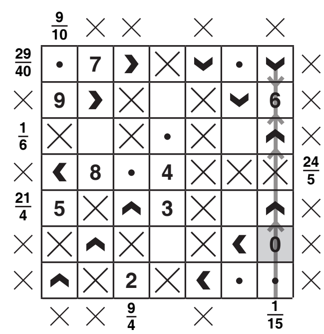
The squares within the big loop cannot contain a 3, nor can any square in the path of $\frac{1}{15} = .0\overline{6}$. Also, we know the path of $\frac{24}{5} = 4.8$ will cross the center square. After eliminating these options, we place the 3 in the only remaining empty square.



The horizontal path through the center square is the only remaining path that does not loop or cross the 3, so this must be the path for $\frac{24}{5} = 4.8$. The 4 cannot go in the first or second square in this path without crossing the path of $\frac{1}{15} = .0\overline{6}$ or entering the big loop. So, the 4 must go in the center square, and the 8 in the square to the left of the 4. Since we do not write leading zeros, we place an $\times$ in the first two squares of the path.



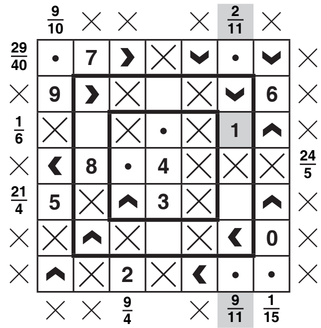
Then, we complete the path for $\frac{1}{15} = .0\overline{6}$ by placing 0 in the only empty square remaining in that path.



The remaining digit is 1, which must go in the third row to complete the path of $\frac{1}{6} = .1\overline{6}$. It cannot go above the 8, because 8 would come before the 1 in the paths for both $\frac{2}{11} = .\overline{18}$ ***and*** $\frac{9}{11} = .\overline{81}$.

Frac-Turn #201, Solution (continued)

So, we place the 1 as shown, and write $\frac{2}{11}$ and $\frac{9}{11}$ in their correct locations.



Each fraction's path correctly traces its decimal form, and we have used all ten digits exactly once. So, we are done.