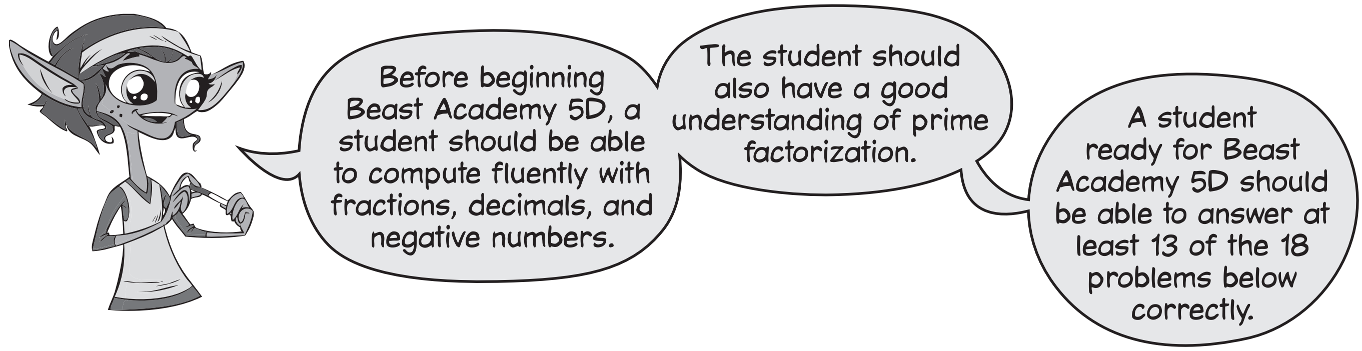
Are you ready for Beast Academy 5D?



Step 1. The student should try to answer every question without a calculator and without help.

Step 2. Check the student's answers using the solutions at the end of this document.

Step 3. The student should be given a second chance on problems that he or she answered incorrectly.

Convert each fraction below into a decimal.

1. $\frac{3}{10} =$ ______

2. $\frac{5}{8} =$ ______

3. $\frac{4}{15} =$ ______

Write each product below as a <u>mixed number</u> in simplest form.

4. $0.85 \cdot 10 =$ ______

5. $\frac{7}{12} \cdot 90 =$ ______

6. $\frac{70}{3} \cdot 0.1 =$ ______

Solve for each variable below. Write fractional answers in simplest form.

7. $\frac{3}{5} = \frac{15}{a}$ $a =$ ________

8. $\frac{c}{10} = \frac{3}{4}$ $c =$ ________

9. $2x + 6 = x$ $x =$ ________

10. $\frac{z+1}{10} + 6 = 3$ $z =$ ________

11. Three more than five times n is 7. What is n? **11.** $n =$ ________

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Fill the blanks to complete each arithmetic sequence below. Write any terms that are not whole numbers as mixed numbers.

12. 39, _____, _____, 53, _____, _____, 67, ...

13. 50, _____, 12, _____, _____, _____, -64, ...

Answer each of the questions below.

14. Fill in the blank below to make a true equation.

$$12 \cdot 14 \cdot 21 \cdot 2 = \boxed{}^{2}$$

15. What is the smallest positive integer that can be multiplied by 0.4 to get an integer result? **15.** ________

16. If we multiply all the terms in sequence below, will the result be positive or negative? **16.** ____________

1, -2, 3, -4, ..., 29, -30, 31.

17. What is the power of 5 in the prime factorization of the number represented by the product below? **17.** ________

$$5 \cdot 10 \cdot 15 \cdot \cdots \cdot 90 \cdot 95 \cdot 100$$

18. The ratio of Edgar's age to Frank's age is 3:6. The sum of their ages is 36. How many years old is Edgar? **18.** ________

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Solutions

1. $\frac{3}{10}$ is 3 tenths. Written as a decimal, 3 tenths is **0.3**.
You may have written **.3**.

2. We wish to convert $\frac{5}{8}$ to an equivalent fraction with a denominator that is a power of 10. Neither 10 nor 100 is a multiple of 8. However, $8\times125=1{,}000$. So, we can convert as shown below.

$$\frac{5}{8} \overset{\times125}{=} \frac{625}{1{,}000}$$

So, $\frac{5}{8}=\frac{625}{1{,}000}$, which is 625 thousandths: **0.625**.
You may have written **.625**.

— or —

Since $\frac{5}{8}$ means $5\div8$, we use long division to write this number as a decimal.

$$\begin{array}{r} 0.6 \\ 8\overline{)5.0} \\ -4.8 \\ \hline 0.2 \end{array} \qquad \begin{array}{r} 0.02 \\ 0.6 \\ 8\overline{)5.0} \\ -4.8 \\ \hline 0.20 \\ -0.16 \\ \hline 0.04 \end{array} \qquad \begin{array}{r} \left.\begin{array}{r} 0.005 \\ 0.02 \\ 0.6 \end{array}\right\} 0.625 \\ 8\overline{)5.0} \\ -4.8 \\ \hline 0.20 \\ -0.16 \\ \hline 0.040 \\ -0.040 \\ \hline 0 \end{array}$$

So, $\frac{5}{8}=$ **0.625**. You may have written **.625**.

3. Since $\frac{4}{15}$ means $4\div15$, we use long division to write this number as a decimal.

$$\begin{array}{r} 0.2 \\ 15\overline{)4.0} \\ -3.0 \\ \hline 1.0 \end{array} \qquad \begin{array}{r} 0.06 \\ 0.2 \\ 15\overline{)4.0} \\ -3.0 \\ \hline 1.00 \\ -0.90 \\ \hline 0.10 \end{array} \qquad \begin{array}{r} 0.006 \\ 0.06 \\ 0.2 \\ 15\overline{)4.0} \\ -3.0 \\ \hline 1.00 \\ -0.90 \\ \hline 0.100 \\ -0.090 \\ \hline 0.010 \end{array}$$

We notice a pattern! Each time we divide after the first step, we get a remainder that is a decimal ending in "10". When we divide these remainders by 15, the quotient is always a decimal ending in 6.

We usually write repeating decimals by drawing a bar over the digits that repeat. So, $\frac{4}{15}=0.2666...=\mathbf{0.2\overline{6}}$.

You may have written $\mathbf{.2\overline{6}}$, or you may have written **0.2666...** with three dots at the end indicating that the 6's continue forever.

4. Multiplying a number by 10 shifts the decimal point one place to the right. So, $0.85\cdot10=8.5$.
Written as a mixed number, $8.5=\mathbf{8\frac{1}{2}}$.

— or —

We start by writing 0.85 as a fraction and simplifying.

$$0.85=\frac{85}{100}=\frac{17}{20}$$

So, $0.85\cdot10=\frac{17}{20}\cdot10$, and we have

$$\frac{17}{\cancel{20}_{2}}\cdot\cancel{10}=\frac{17}{2}=\mathbf{8\frac{1}{2}}.$$

5. $\frac{7}{12}\cdot90=\frac{630}{12}=\frac{105}{2}=\mathbf{52\frac{1}{2}}$.

— or —

$\frac{7}{12}\cdot90=\frac{7}{\cancel{12}_{2}}\cdot\cancel{90}^{15}=\frac{105}{2}=\mathbf{52\frac{1}{2}}$.

6. $0.1=\frac{1}{10}$. So, $\frac{70}{3}\cdot0.1=\frac{70}{3}\cdot\frac{1}{10}$, and we have

$$\frac{70}{3}\cdot\frac{1}{10}=\frac{\cancel{70}^{7}}{3}\cdot\frac{1}{\cancel{10}}=\frac{7}{3}=\mathbf{2\frac{1}{3}}.$$

7. We multiply the numerator and denominator of $\frac{3}{5}$ by 5 to get an equivalent fraction with numerator 15.

$$\frac{3}{5} \overset{\times5}{=} \frac{15}{25}$$

Since $\frac{3}{5}=\frac{15}{a}$ and $\frac{3}{5}=\frac{15}{25}$, we have $a=\mathbf{25}$.

— or —

We multiply both sides by $5a$ and then solve for a as shown below.

$$\begin{aligned} \frac{3}{5}&=\frac{15}{a} \\ \frac{3}{\cancel{5}}\cdot\cancel{5}a&=\frac{15}{\cancel{a}}\cdot5\cancel{a} \\ 3\cdot a&=15\cdot5 \\ 3a&=75 \\ a&=\frac{75}{3} \\ a&=\mathbf{25}. \end{aligned}$$

8. We multiply both sides by 10 and then solve for c as shown below.

$$\begin{aligned} \frac{c}{10}&=\frac{3}{4} \\ \frac{c}{\cancel{10}}\cdot\cancel{10}&=\frac{3}{\cancel{4}_{2}}\cdot\cancel{10}^{5} \\ c&=\mathbf{\frac{15}{2}} \\ c&=\mathbf{7\frac{1}{2}}. \end{aligned}$$

You may have instead written **7.5**.

9. Subtracting x from both sides of the equation gives

$$x+6=0$$

Then, subtracting 6 from both sides gives

$$x=\mathbf{-6}.$$

10. Subtracting 6 from both sides of the equation gives

$$\frac{z+1}{10}=-3.$$

Then, multiplying both sides by 10 gives

$$\begin{aligned} \frac{z+1}{10}&=-3 \\ \frac{z+1}{\cancel{10}}\cdot\cancel{10}&=-3\cdot10 \\ z+1&=-30. \end{aligned}$$

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Finally, subtracting 1 from both sides gives

$$z = \mathbf{-31}.$$

11. We first translate the words into an equation. "Five times n" is $5n$, so "three more than 5 times n" is $5n+3$.

Therefore, the equation given by the sentence is $5n+3=7$.

Subtracting 3 from both sides of this equation gives

$$5n=4.$$

Dividing both sides by 5 gives $n=\mathbf{\frac{4}{5}}$. You may have instead written $\frac{4}{5}$ as a decimal: **0.8** or **.8**.

12. In an arithmetic sequence, the same amount is always added from one term to the next. The amount that is added to get each next term is called the common difference.

The sequence has 1st term 39 and 4th term 53, so the common difference is $\frac{53-39}{3}=\frac{14}{3}=4\frac{2}{3}$. We use this to fill in the remaining blanks as shown.

$$39 \xrightarrow{+4\frac{2}{3}} \mathbf{43\tfrac{2}{3}} \xrightarrow{+4\frac{2}{3}} \mathbf{48\tfrac{1}{3}} \xrightarrow{+4\frac{2}{3}} 53 \xrightarrow{+4\frac{2}{3}} \mathbf{57\tfrac{2}{3}} \xrightarrow{+4\frac{2}{3}} \mathbf{62\tfrac{1}{3}} \xrightarrow{+4\frac{2}{3}} 67, \ldots$$

13. The sequence has 1st term 50 and 3rd term 12, so the common difference is $\frac{12-50}{2}=\frac{-38}{2}=-19$. We use this to fill in the remaining blanks as shown.

$$50 \xrightarrow{-19} \mathbf{31} \xrightarrow{-19} 12 \xrightarrow{-19} \mathbf{-7} \xrightarrow{-19} \mathbf{-26} \xrightarrow{-19} \mathbf{-45} \xrightarrow{-19} -64, \ldots$$

14. We consider the prime factorization of the expression on the left.

$$\begin{aligned} 12\cdot 14\cdot 21\cdot 2 &= (2\cdot 2\cdot 3)\cdot(2\cdot 7)\cdot(3\cdot 7)\cdot 2 \\ &= 2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 7\cdot 7. \end{aligned}$$

We can split these prime factors into two equal groups:

$$\begin{aligned} 2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 7\cdot 7 &= (2\cdot 2\cdot 3\cdot 7)(2\cdot 2\cdot 3\cdot 7) \\ &= (2\cdot 2\cdot 3\cdot 7)^2. \end{aligned}$$

Since $2\cdot 2\cdot 3\cdot 7=84$, we have

$$12\cdot 14\cdot 21\cdot 2=\boxed{\mathbf{84}}^2.$$

15. Multiplying 0.4 by 10 moves the decimal point in 0.4 one places to the right, giving the integer 4:

$$0.4\cdot 10=4.$$

Both 10 and 4 are divisible by 2. So, multiplying 0.4 by $\frac{1}{2}$ of 10 will give a result that is $\frac{1}{2}$ of 4.

$\frac{1}{2}$ of 10 is 5, and $\frac{1}{2}$ of 4 is 2. So, we have

$$0.4\cdot 5=2.$$

5 and 2 are not both divisible by an integer greater than 1, so **5** is the smallest positive integer we can multiply by 0.4 to get an integer result.

— or —

We write this decimal as a fraction in simplest form, $0.4=\frac{4}{10}=\frac{2}{5}$.

We know that $\frac{2}{5}\cdot 5=2$.

If we multiply $\frac{2}{5}$ by any positive integer less than 5, then the 5 in the denominator of $\frac{2}{5}$ will not cancel and the result will not be an integer.

So, **5** is the smallest positive integer we can multiply by 0.4 to get an integer result.

16. The sign of the answer is determined by the number of negatives in the product.

The negative numbers in this sequence are -2, -4, -6, ..., -26, -28, -30.

To count the number of negatives in the list above, we divide each of the numbers by -2. This gives us

$$1, 2, 3, \ldots, 13, 14, 15.$$

Therefore, there are 15 negative numbers in this list, so there are 15 negative numbers in the original sequence.

Any product with an odd number of negatives (and no zeros) is **negative**.

17. There are 20 numbers in the given product, each of which contributes 1 factor of 5 to the prime factorization:

$$(1\cdot 5), (2\cdot 5), (3\cdot 5), \ldots, (18\cdot 5), (19\cdot 5), (20\cdot 5).$$

However, multiples of $25=5^2$ have two 5's in their prime factorizations. So, these numbers contribute 2 factors of 5 to the prime factorization. There are four multiples of 25 in this product, which each contribute one more 5 to the prime factorization: 25, 50, 75, and 100.

$5^3=125$ is greater than 100, so none of the numbers in the given product contribute 3 or more factors of 5 to the prime factorization.

So, the exponent of 5 in the prime factorization of the product is $20+4=24$. Therefore, the power of 5 in the prime factorization of $5\cdot 10\cdot 15\cdot\cdots\cdot 90\cdot 95\cdot 100$ is $\mathbf{5^{24}}$.

18. From the given ratio, we know that for some value of x, Edgar is $3x$ years old and Frank is $6x$ years old. The sum of their ages is $3x+6x=9x$.

Since the sum of their ages is 36, we write an equation:

$$9x=36.$$

Dividing both sides by 9 gives $x=4$.

Therefore, Edgar is $3\cdot 4=$ **12** years old.

— or —

We first simplify the ratio:

$$3:6=1:2.$$

Therefore, for some value of y, Edgar is y years old, and Frank is $2y$ years old. The sum of their ages is $y+2y=3y$.

Since the sum of their ages is 36, we write an equation:

$$3y=36.$$

Dividing both sides by 3 gives $y=12$. Edgar is **12** years old.